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The Role of Large Scalar Amplitudes in High-Temperature Baryon-Number Violation

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Abstract

Using the Cline-Raby result relating the high-temperature rate for baryon-number violation to zero-temperature baryon-number-violating amplitudes, I consider the effect of the large tree-level scalar amplitudes noted by Cornwall and Goldberg on the high-temperature rate for baryon-number violation. These effects may suppress the high-temperature rate to unobservable levels.

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Perturbative amplitudes in the standard $SU(3)\times SU(2)\times U(1)$ model of particle interactions preserve baryon number and separate lepton numbers for each of the three generations. Baryon-number violation can arise in perturbation theory, through higher-dimension operators descended from unifying groups; or it can arise through non-perturbative effects, because the baryon current has an $SU(2)$ anomaly. As shown by 't Hooft, these effects are dominated in weak coupling by stationary points of the Euclidean action, the instantons, which have an action proportional to $1/g^2$. These lead to an exponential suppression in the low-energy zero-temperature rate of $\exp(-4\pi/\alpha_w)$, making these effects unobservably small. At high temperature, however, various authors [1, 2] have argued that this exponential suppression may be overcome.

Recently, an even more remarkable possibility was suggested by the calculations of Ringwald [3], Espinosa [4], and McLerran, Vainshtein, and Voloshin [5]. These authors calculate the instanton contributions to $\Delta B\neq 0$ processes with the minimal number of fermions required eat up the instanton zero modes, and with a large number of gauge bosons or Higgs scalars. From these calculations one would obtain

$$\mathcal{A}_{2\to n}\sim e^{-2\pi/\alpha_w}\Gamma(n)\beta^n\left(\frac{g}{M_W}\right)^{n-2} \tag{1}$$

for the amplitude of the $2\to n$ process, where β is a number of $O(1)$. (I will ignore all distinguishable dependence on the number of fermions, as well as the distinction between gauge bosons and Higgs particles.) Squaring this and integrating over relativistic phase space yields

$$\sigma_{2\to n}\sim e^{-2\pi/\alpha_w}\frac{1}{n!}\alpha_w^{n-2}s^{-1}\beta^{2n}\left(\frac{s}{M_W^2}\right)^{n-2} \tag{2}$$

If we sum over n (in practice the sum would be cut off at $n\sim E/M_W\sim E/m_H$), we obtain a rapidly growing total cross section,

$$\sigma_{2\to\mathrm{all}}\sim\exp\left[-\frac{2\pi}{\alpha_w}+\beta\frac{\alpha_w s}{M_W^2}\right]\frac{1}{s}\equiv\frac{1}{s}\exp\left[-\frac{2\pi}{\alpha_w}\xi(s)\right] \tag{3}$$

If s is sufficiently large, it would appear again that the exponential suppression could be overcome.

This calculation (or more precisely, the applicability of the calculation to the scattering of high-energy particles) has been criticized by several different groups. The basic observation is that there is a scale-size mismatch in the problem: the field configurations that dominate the results of Ringwald and Espinosa are large, of size $\sqrt{n}/v$, where n is the number of final-state bosons, and v is the Higgs expectation value. On the other hand, one expects [6] the configurations relevant in high-energy fixed angle scattering to have size of order $1/\sqrt{s}$, that is the size scales inversely with the energy of the incoming particles. In the Minkowski-space calculations of Banks et al. [7], this is reflected in an exponential suppression coming from the wavefunction overlap of the high-energy initial state and the classical configuration which sits at the top of the tunnelling barrier. (See also Singleton et al. [8].)

In Euclidean space, a partial resolution of this issue may be found in the calculations of Aoki and Mazur [9]. The relevant Euclidean and Minkowski configurations are actually closely related (the constant t slices of the Minkowski configurations corresponding to the minimum barrier height are just the constant Euclidean τ slices of the instanton), so one should expect to gain some insight into the relevant Euclidean configurations through heuristic Minkowski-space arguments of Mueller [6]. In the usual sorts of tunnelling calculations, where one is investigating (say) the decay of a false vacuum, it is appropriate to integrate over all locations of the center of the 'instanton', as well as all scale sizes, because the vacuum does not possess any special scale, at least on distances much shorter than the infrared cut-off. In the scattering calculation, the situation is different. Suppose we consider a fixed-angle scattering of two high-energy wave packets (by fixed angle I mean that $\sum_{\text{final}} |\mathbf{p}_{\mathrm{T}}| \sim \sqrt{s}$). Then the size of the interaction region is of order $1/\sqrt{s}$, and only field configurations which have significant variation on a scale of $1/\sqrt{s}$ near the interaction region should be expected to contribute. Thus we should expect the presence of the external fields to determine which subset of 'instantons' are relevant to the scattering problem.

That is what happens in the calculations of Aoki and Mazur. The basic point is rather than trying to estimate the Green function for a baryon number violating process with some number of external scalars (or gauge bosons),

$$G(\{x_i, y_i, z_i\}) = \int [DA_\mu][D\phi][D\psi][D\overline{\psi}]\, e^{-S[A_\mu,\phi,\overline{\psi}]} \prod \left\{ \psi^{(0)}(x_i), A_\mu(y_i), \phi(z_i) \right\} \qquad (4)$$

by expanding around the saddle point of the exponential of the action, one should estimate it by finding the saddle point of the entire integrand, including external sources. The classical action S_{cl} is independent of the location of the instanton, and up to corrections coming from the Higgs VEV, independent of the size of the instanton (that is why these are collective coordinates). However, the same is not true of the external sources, and thus the effective action in the presence of the sources, $S_{\mathrm{cl}} + \sum \log \left\{ \psi^{(0)}(x_i)/\Lambda^{3/2}, A_\mu(y_i)/\Lambda, \phi(z_i)/\Lambda \right\}$, will yield variational equations that determine the collective coordinates of the instanton. In particular, the scale size of the instanton scales inversely with the incoming energy. Numerical estimates in a Minkowski-space calculation by Cornwall [10], and by Mazur [11] suggest that while the cross-section does grow at low energy, $\xi(s)$ in equation (3) is always larger than 1/3 to 1/2.

It thus appears likely that $2 \to n$ baryon-number violating scattering cross sections remain small. However, the reasoning that suggests that equation (2) is not applicable as an estimate of high-energy scattering also suggests that the analogous equation *is* valid as an estimate of an $n \to n$ process when *all* external particles are soft (energies just enough larger than M_W to avoid non-relativistic phase space suppression). In this case, when all sources are soft, the relevant field configurations are expected to be large, and the exponential suppression revealed by Banks et al. [7] should disappear.

One can then use the result of Cline and Raby [12] to apply intuitions (or rough results) about zero-temperature scattering amplitudes to the high-temperature problem. These authors show that the rate for baryon-number violation (per unit volume) at high temperature, $\gamma_{\Delta B}$, is related to a sum over all possible $n \to m$ zero-temperature scattering processes, weighted by appropriate Boltzmann factors for the incoming particles,

$$\gamma_{\Delta B} = \frac{1}{2}\sum_{n=1}^{\infty}\int\frac{1}{n!}\left(\prod_{j=1}^{n}\frac{d^3p_j}{(2\pi)^3 2E_j}\right)\sum_{l=1}^{\infty}\left(\Delta B_{n\to l}\right)^2\sigma_{n\to l}\left(\{p_j\}_{j=1}^{n}\right)\,\exp\left[-\frac{1}{k_BT}\sum_{j=1}^{n}E_j\right] \tag{5}$$

An exponentially small $2 \to n$ cross section is thus consistent with a substantial rate for baryon-number-violating processes at high temperature, so long as the rate for $n \to m$ processes, with n and m large, is not suppressed; and the calculations described earlier suggest that this indeed may be the case. This much is consistent with earlier investigations of the high-temperature question.

However, baryon-number-*violating* processes are not the only ones that get large when the total multiplicity gets large. Cornwall [10, 13] and Goldberg [14] have noted that ordinary tree-level perturbation theory for a scalar ϕ^4 theory yields a prediction for factorially growing cross sections for n-particle processes. (The case of gauge bosons is less clear, because in a gauge theory the possibility exists of finding cancellations between different Feynman diagrams. Thus I will henceforth restrict attention to amplitudes containing mostly scalars in the final state.) Cornwall finds an approximate solution to the classical field equations in the presence of many soft sources, and then estimates the amplitude by functionally differentiating with respect to the source. One then finds for a $n \to m$ amplitude,

$$\mathcal{A}_{n\to m} \sim \left(\beta'\sqrt{\lambda}\right)^{n+m-2}\Gamma(n)\Gamma(m)(n+m)^{n+m-4}E_{\rm tot}^{4-n-m} \tag{6}$$

where β' is another constant of $O(1)$. Although no tunnelling is involved in these processes, there is nonetheless a great deal of similarity to the calculation of baryon-number violation; both proceed via classical configurations.

Once again, crossing to $2 \to n$ amplitudes involves large analytic continuations in the momentum invariants. The scale-size mismatch suggests that a softer function of n will enter the behavior of this amplitude, though it is not clear precisely how to estimate it. (Alternatively, a suppression along the lines of Banks et al. [7] may arise from the small overlap of the initial high-energy state, and the classical configuration in the intermediate state which then evolves to yield the n-particle final state.) The interpretation of Goldberg's results would then be to indicate a failure of the saddle-point expansion about the usual perturbative vacuum $\phi = 0$, because the saddle point will have been shifted substantially by the external sources. This would be analogous to what happens in the Aoki-Mazur instanton calculation.

Whatever the resolution in the case of $2 \to n$ processes, these arguments do seem trustworthy (just as in the case of baryon-number-violating amplitudes) for processes with large numbers of particles in both the initial and final states, where all the energies of the external particles are small compared to the total center-of-mass energy.

These are very similar to the amplitudes for baryon-number violating processes — except that the latter still contain an over-all factor of $\exp(-2\pi/\alpha_w)$. (We are considering $E_{\rm tot}/(n+m) \sim M_W \cdot O(1)$; note also that for $n \simeq m$, $\Gamma(n)\Gamma(m) \simeq \Gamma(n+m)\cdot(O(1))^{n+m}$.) In both cases, the amplitudes, taken at face value, will eventually violate limits dictated by unitarity. Of course, the full theory is smarter than that; this merely indicates the breakdown of the approximations used in deriving the amplitudes. How will these amplitudes unitarize?

The presence of large amplitudes in $n \to n$ processes can be interpreted as giving rise to large 'final-state' interactions of the outgoing particles in a process producing a large number of scalars. Indeed, one may imagine [13] that unitarity is recovered simply by iterating the $n \to n$ amplitude many times,

$$
\begin{aligned}
&\mathcal{A}^{\rm full}_{\Delta B=0}\left(\{p_i\}_{\rm in},\{q_i\}_{\rm out}\right) = \\
&\qquad \mathcal{A}^{\rm naive}_{\Delta B=0}\left(\{p_i\}_{\rm in},\{q_i\}_{\rm out}\right) + \sum_n \int \frac{d^4 k_{j=1\ldots n}}{(2\pi)^4}\, \mathcal{A}^{\rm naive}_{\Delta B=0}\left(\{p_i\}_{\rm in},\{k_j\}\right) \mathcal{A}^{\rm naive}_{\Delta B=0}\left(\{k_j\},\{q_i\}_{\rm out}\right) \\
&\qquad + \sum_{n,n'} \int \frac{d^4 k_{j=1\ldots n}}{(2\pi)^4}\, \frac{d^4 k'_{j'=1\ldots n'}}{(2\pi)^4} \\
&\qquad\quad \times \mathcal{A}^{\rm naive}_{\Delta B=0}\left(\{p_i\}_{\rm in},\{k_j\}\right) \mathcal{A}^{\rm naive}_{\Delta B=0}\left(\{k_j\},\{k'_{j'}\}\right) \mathcal{A}^{\rm naive}_{\Delta B=0}\left(\{k'_{j'}\},\{q_i\}_{\rm out}\right) + \cdots
\end{aligned}
\tag{7}
$$

where $\mathcal{A}^{\rm naive}_{\Delta B=0}$ is given by equation (6). If we assume that these integrals are dominated by physical intermediate states, and sum up the series, we arrive at the model of unitarization employed by

Cornwall [10,13],

$$\mathcal{A}^{\text{full}}_{\Delta B=0} = \mathcal{A}^{\text{naive}}_{\Delta B=0}\rho^{1/2}\left(1 - i\rho^{1/2}\mathcal{A}^{\text{naive}}_{\Delta B=0}\rho^{1/2}\right)^{-1}\rho^{-1/2} \tag{8}$$

where matrix notation is implied $\left(\mathcal{A}^{\text{naive}}_{\Delta B=0}\right)_{nm} = \mathcal{A}^{\text{naive}}_{\Delta B=0;n\to m}$, $\mathcal{A}^{\text{naive}}_{\Delta B=0}$ is taken to be real, and $\rho_{nm} = \delta_{nm}(E/4\pi)^{n-4}/\Gamma(n)^3$. As noted there, this form automatically satisfies the form of the optical theorem appropriate for the approximate estimates used in this paper,

$$\text{Im}\,\mathcal{A}^{\text{full}}_{\Delta B=0} = \mathcal{A}^{\text{full}}_{\Delta B=0}\,\rho\,\mathcal{A}^{\text{full}}_{\Delta B=0}{}^{\dagger} \tag{9}$$

Of course, one can iterate the baryon-number-conserving $n \to m$ processes not only in the final states produced by such processes themselves, but also in the final states produced by baryon-number-*violating* processes. This will lead to a form for these amplitudes,

$$\mathcal{A}^{\text{full}}_{\Delta B\neq 0} = \mathcal{A}^{\text{naive}}_{\Delta B\neq 0}\rho^{1/2}\left(1 - i\rho^{1/2}\mathcal{A}^{\text{naive}}_{\Delta B=0}\rho^{1/2}\right)^{-1}\rho^{-1/2} \tag{10}$$

The key point is that because the baryon-number-conserving processes lack the $\exp(-2\pi/\alpha_w)$ suppression, they will reach the unitarity limit, at which these iteration effects become important, before the baryon-number violating processes do. At this point, it is appropriate to iterate the former, but to treat the latter perturbatively. The ratio of the two processes is then given by

$$\frac{\left(\mathcal{A}^{\text{full}}_{\Delta B\neq 0}\right)_{nm}}{\left(\mathcal{A}^{\text{full}}_{\Delta B=0}\right)_{nm}} \sim \frac{\left(\mathcal{A}^{\text{naive}}_{\Delta B\neq 0}\right)_{nm}}{\left(\mathcal{A}^{\text{naive}}_{\Delta B=0}\right)_{nm}} \simeq \exp\left[-\frac{2\pi}{\alpha_w}\right]\left(\frac{\beta}{\beta'}\right)^{n+m} \tag{11}$$

since the latter approximation is true for all n and m, and since λ is $O(g^2)$. Once the leading channel hits the unitarity limit, further growth in any process is essentially cut off; the ratio of the baryon-number-violating amplitude, to that of the baryon-number-conserving amplitude, remains fixed — and exponentially small. It seems implausible that this difference could be overcome through the ratio β'/β; for the standard model, the relevant $n + m \sim 1/\alpha_w$, so we would need $\beta'/\beta < 10^{-3}$ (recall that β and β' are independent of coupling constants).

This expectation is consistent with the behavior of higher-dimension operators in walking technicolor theories [15]. In these theories, iteration of a technicolor gauge interaction whose strength is just below the critical value for chiral symmetry-breaking induces large anomalous dimensions in higher-dimension operators (for example, two-technifermion two-quark operators responsible for quark masses), which reduce the rate at which such operators grow strong as the momentum transfer through them is increased, and thus increase the analog of the unitarity-violating scale.

In view of the Cline-Raby result, equation (5), the recovery of exponential suppression in zero-temperature many-body baryon-number-violating processes in turn implies that the rate for high-temperature baryon-number violation involving associated scalar production remains exponentially small, in spite of the results of refs. [1,2]. As noted above, with our present knowledge, these arguments can be applied only to those baryon-number-violating amplitudes with a large number of scalars in the final state. If the behavior found by Cornwall and Goldberg is also characteristic of gauge theories, the arguments given above would apply to all the amplitudes considered in refs. [1,2], and one would find the re-emergence of an exponential suppression of high-temperature baryon-number violation.

The corresponding argument in the language of thermal diffusion over the barrier is obscure, but the preceeding arguments suggest that it is not appropriate to reduce the problem to a one-dimensional barrier question in a $\Delta B \neq 0$ direction, because ordinary perturbative fluctuations are in effect strongly coupled (possibly due to the emergence of non-trivial baryon-number-free configurations).

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